Universal Inverse Power Law Distribution for Indian Region Rainfall

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Abstract

Space-time fluctuations of meteorological parameters exhibit selfsimilar fractal fluctuations. Fractal space-time fluctuations are generic to dynamical systems in nature such as fluid flows, spread of diseases, heart beat pattern, etc. A general systems theory developed by the author predicts universal inverse power law form incorporating the golden mean for the fractal fluctuations. The model predicted distribution is in close agreement with observed fractal fluctuations of all size scales in the monthly total Indian region rainfall for the 141 year period 1871 to 2011.

Keywords: *Fractal fluctuations; Universal inverse power law; Indian region rainfall*

1. Introduction

Dynamical systems such as fluid flows, heart beat patterns, spread of infectious diseases, etc., exhibit selfsimilar, i.e., a zig-zag pattern of successive increases followed by decreases of all scales identified as fractal fluctuations. Fractal fluctuations signify non-local connections, i.e., long-range correlations in space and time. Lovejoy and Schertzer (2012) have done pioneering work during the last 30 years to identify conclusively the selfsimilar fractal nature of fluctuations in meteorological parameters. The Gaussian probability distribution used widely for analysis and description of large data sets underestimates the probabilities of occurrence of extreme events such as stock market crashes, earthquakes, heavy rainfall, etc. The assumptions underlying the normal distribution such as fixed mean and standard deviation, independence of data, are not valid for real world fractal data sets exhibiting a scale-free power law distribution with fat tails (Selvam, 2009). There is now urgent need to incorporate newly identified fractal concepts in standard meteorological theory for realistic simulation and prediction of atmospheric flows. The author has developed a general systems theory model (Selvam, 2012a, Selvam, 2012b, Selvam, 2013) for fractal fluctuations in dynamical systems. The model predicts universal inverse power law form incorporating the golden mean ($\tau \approx 1.618$) for the probability distribution of amplitudes of fractal fluctuations. The model predictions are in agreement with monthly total rainfall over the Indian region for the 141-year period 1871-2011. The paper is arranged as follows. Section 2 gives a brief summary of the general systems theory model predictions for fractal fluctuations in dynamical systems. Section 3 gives details of data and analysis techniques. A brief discussion of results in Section 4 is followed by Conclusions in Section 5.

2. General Systems Theory for Fractal Fluctuations

Power (variance) spectra of fractal fluctuations exhibit inverse power law form f^{α} where f is the frequency (or wavelength of the eddies) and α the exponent indicating (i) selfsimilar fractal fluctuations result from the coexistence of a continuum of eddies (waves) (ii) fractal fluctuations exhibit long-range space-time correlations since the amplitudes of larger and smaller size eddies are related to each other by the scale factor α alone independent of other characteristics of the eddies.

The general systems theory model (Selvam, 1990, 2007, 2012a, 2012b, 2013) is based on the above observational fact that fractal fluctuations signify an underlying eddy continuum. The model is based on the simple concept that large eddies result from successive space-time integration of enclosed small-scale fluctuations (eddies) analogous to Townsend's (1956) concept that large eddies are envelopes enclosing smaller scale eddies. The model predictions are

i. Starting from unit primary eddy (radius *r*), the successive stages of large eddy (radius *R*) growth is associated with scale (length) ratio *z* equal to *R*/*r* and forms an eddy continuum which can be resolved into an overall logarithmic spiral trajectory tracing the quasiperiodic Penrose tiling pattern identified as quasicrystalline structure in condensed matter physics. Starting with unit primary eddy, successive stages of large eddy growth is associated with scale ratio z = to 1, 2, 3, etc. The primary eddy growth region is $z = 0$ to 1.

ii. The probability distribution of amplitude and variance (square of amplitude) of fractal fluctuations (space/time series) when plotted with respect to normalized standard deviation σ equal to mean/standard deviation follow the same inverse power law form *P*.

iii. For the range of normalized deviation σ values $\sigma \geq 1$ and $\sigma \leq -1$, the probability distribution $P = \tau^{-4\sigma}$.

iv. Normalised deviation σ ranging from -1 to +1 corresponds to the primary eddy growth region. In this region the probability *P* is shown to be equal to $P = \tau^{-4k}$ where $k = \sqrt{\frac{\pi}{2z}}$ is the steady state fractional volume dilution *k* of the growing primary eddy by internal smaller scale eddy mixing (Selvam, 2013).

v. The model predicted universal inverse power law distribution is very close to the statistical normal distribution for normalized deviation σ values less than 2 and exhibits a long fat tail for σ values more than 2, i.e., extreme events have a higher probability of occurrence than that predicted by statistical normal distribution as found in practice. The statistical normal distribution and the model predicted universal inverse power law distribution are shown in Fig.1 (Selvam, 2013).

vi. Fractal fluctuations signify quantumlike chaos since the property that the additive amplitudes of eddies when squared represent the probability densities is exhibited by the subatomic dynamics of quantum systems such as the electron or photon.

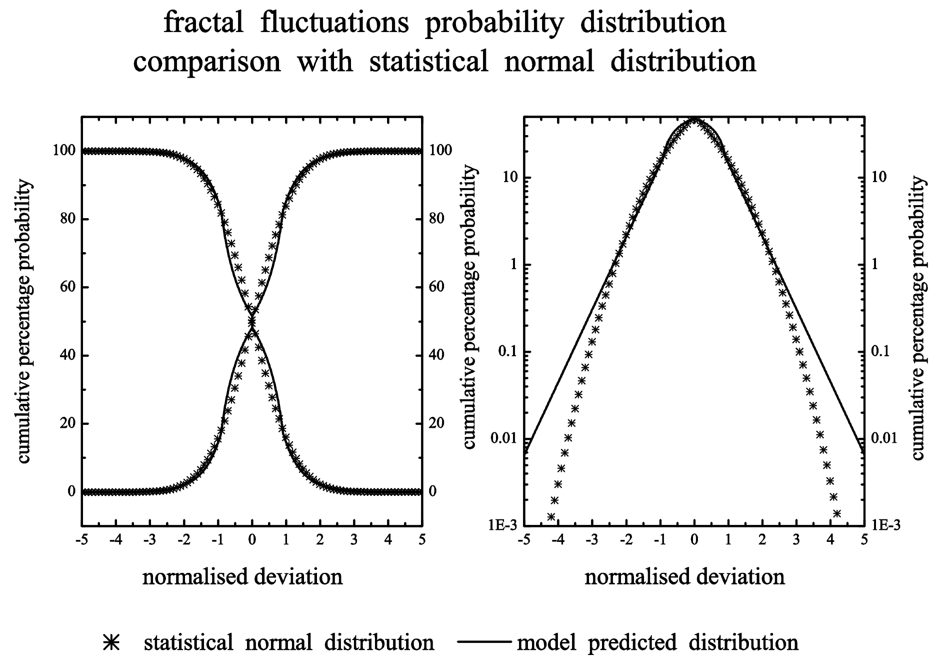

Fig. 1: Model predicted probability distribution *P* along with the corresponding statistical normal distribution with probability values plotted on linear and logarithmic scales respectively on the left and right hand sides.

3. Data

Monthly (January to December) Data (upto 1 decimal in mm) for the 141 year period (1871-2011) for the eight meteorological subdivisions of India (i) All-India (ii) Homogeneous (iii) Core-Monsoon (iv) Northwest (v) West Central (vi) Central Northeast (vii) Northeast (viii) Peninsular were obtained from ftp://www.tropmet.res.in/pub/data/rain/iitm-regionrf.txt and used for the study.

3.1 Analyses and results

Each data set was represented as the frequency of occurrence $f(i)$ in a suitable number n of class intervals $x(i)$, i=1, n covering the range of values from *minimum* to the *maximum* in the data set. The class interval $x(i)$ represents dataset values in the range $x(i) \pm \Delta x$, where Δx is a constant. The average *av* and standard deviation *sd* for the data set is computed as

$$av = \frac{\sum_{1}^{n}\left[x(i)\times f(i)\right]}{\sum_{1}^{n} f(i)}$$

$$sd = \frac{\sum_{1}^{n}\left\{\left[x(i)-av\right]^{2}\times f(i)\right\}}{\sum_{1}^{n} f(i)}$$

The monthwise average and standard deviation values of rainfall for the 141-year period (1871-2011) for the eight meteorological subdivisions of India are given in Fig. 2.

The *normalized deviation* σ values for class intervals $t(i)$ were then computed as

$$\sigma(i) = \frac{x(i)-av}{sd}$$

The cumulative percentage probabilities of occurrence $cmax(i)$ and $cmin(i)$ were then computed starting respectively from the maximum (i=n) and minimum (i=1) class interval values as follows.

$$cmax(i) = \frac{\sum_{n}^{i}\left[x(i)\times f(i)\right]}{\sum_{1}^{n}\left[x(i)\times f(i)\right]}\times 100.0$$

$$cmin(i) = \frac{\sum_{1}^{i}\left[x(i)\times f(i)\right]}{\sum_{1}^{n}\left[x(i)\times f(i)\right]}\times 100.0$$

The 12-month average and standard deviation of cumulative percentage probability values $cmax(i)$ and $cmin(i)$ were computed for each meteorological subdivision and plotted with respect to corresponding *normalized deviation* $t(i)$ values with logarithmic scale for the probability axis (Fig. 3) along with model predicted universal inverse power law distribution. There is a close correspondence between model predicted and observed probability distributions of amplitudes of fractal fluctuations of all size scales in Indian region rainfall.

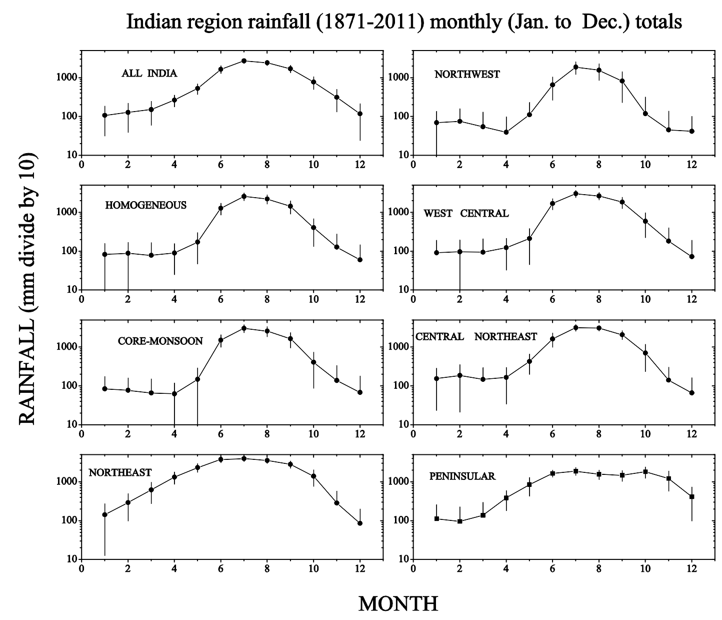

Fig. 2: The monthwise average and standard deviation values of rainfall for the 141-year period (1871-2011) for the eight meteorological subdivisions of India

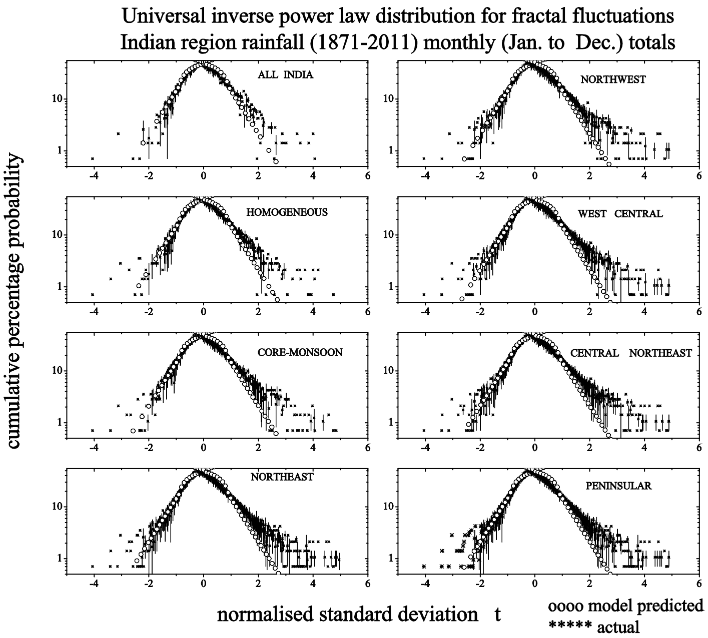

Fig. 3: The 12-month average and standard deviation of cumulative percentage probability values for each meteorological subdivision and plotted with respect to corresponding *normalized deviation* $t(i)$ values with logarithmic scale for the probability axis along with model predicted universal inverse power law distribution.

4. Discussion

The probability distribution P of amplitudes of fractal fluctuations in Indian region rainfall for fluctuations of all size scales closely follows the general systems theory model predicted universal inverse power law distribution $P = \tau^{-4\sigma}$ where τ is the golden mean (≈ 1.618) and σ the normalized deviation equal to mean/standard deviation. The model predicted distribution is close to the observed distribution particularly for the normalized deviation σ values greater than 2 which correspond to extreme events with higher probability of occurrence than that predicted by the statistical normal distribution.

Inverse power law distribution for fractal fluctuations implies long-range space-time correlations manifested as memory or persistence in the space-time variability of the meteorological parameter such as rainfall, temperature, etc. Kantelhardt et al. (2006) state that the persistence analysis of river flows and precipitation has been initiated, about half a century ago, by H. E. Hurst, who found that runoff records from various rivers exhibit ‘‘long-range statistical dependencies’’ (Hurst, 1951). Later, similar long-term correlated fluctuation behavior has also been reported for many other geophysical records including temperature and precipitation data (Kantelhardt et al., 2006). Characterizing and understanding the persistence of wet and dry conditions in the distant past gives new perspectives on contemporary climate change and its causes (Bunde et al., 2013).

5. Conclusion

A general systems theory model developed by the author predicts universal inverse power law form incorporating the golden mean for the fractal fluctuations. The model predicted distribution is in close agreement with observed fractal fluctuations of all size scales in the monthly total Indian region rainfall for the 141 year period 1871 to 2011.

Acknowledgement

The author is grateful to DR. A. S. R. Murty for encouragement during the course of the study.

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